Mitigation of Ionospheric Effects on High Frequency Surface Wave Radar

Dr. Ryan J. Riddolls
Defence R&D Canada - Ottawa
3701 Carling Avenue
Ottawa, Ontario K1A 0Z4
Canada
ryan.riddolls@drdc-rddc.gc.ca

ABSTRACT

High Frequency Surface Wave Radar (HFSWR) takes advantage of the diffraction of electromagnetic waves over the conducting ocean surface to detect, locate, and track surface vessels beyond the line-of-sight horizon of the earth. However, long-range surface vessel detection is often confounded by radar clutter comprising echoes from the ionospheric plasma. In this paper, we characterize the angular spectrum of these reflections, and from this information deduce the signal-to-clutter processing gain that can be realized by various adaptive receive antenna array configurations. In particular, it is shown that there is advantage to be realized by sampling ionospheric echoes with a planar two-dimensional array rather than a conventional linear one-dimensional HFSWR array. Using a planar array, the radar can distinguish high-elevation ionospheric clutter signals from low-elevation surface target echoes.

1 ADAPTIVE ARRAYS

Ionospheric reflection in HFSWR applications complicates the detection of weak target echoes. In particular, during nighttime Spread F conditions, ionospheric clutter can occupy most of the far ranges in the radar range-Doppler search plane. Adaptive antenna array techniques can be used to increase the ratio of surface vessel echo power to ionospheric clutter power in cluttered radar ranges.

We consider a surface vessel radar echo consisting of a plane wave of amplitude x that is received by an array of antenna elements arranged in an arbitrary configuration. The amplitude and phase response of the individual sensors to the plane wave is captured by an array manifold vector $\mathbf{v}$. In addition to the plane wave signal, the sensors are corrupted by a "noise" vector $\mathbf{n}$ that is dominated by ionospheric clutter signals. Thus the signal measured by the array can be written as $\mathbf{y} = \mathbf{v}x + \mathbf{n}$.

We assume the radar signals are narrowband such that they can be described by complex amplitudes. We devise a linear estimator for x that consists of summing the measured signals according to a "weight" function $\mathbf{w}$. This estimator is denoted as $\hat{x} = \mathbf{w}^H\mathbf{y}$, where H is the complex conjugate transpose. In order to minimize the variance of the expected value $E(\hat{x})$ under the constraint of no distortion $E(\hat{x}) = E(x)$, the optimal choice of weights in this linear estimator is given by [1]:

$$\mathbf{w}^H = \frac{\mathbf{v}^H\mathbf{R}_n^{-1}}{\mathbf{v}^H\mathbf{R}_n^{-1}\mathbf{v}}, \tag{1}$$

where $\mathbf{R}_n = E(\mathbf{n}\mathbf{n}^H)$ is the noise covariance matrix. This estimator is an adaptive beamformer, providing constructive interference in the target direction and destructive interference in noise directions.

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This beamformer is distortionless, so it provides no improvement in target signal power. All SNR improvement arises from reducing noise power associated with the ionospheric clutter. At the input to the beamformer, the noise power received by individual antenna elements is given by the diagonal entries of $\mathbf{R}_n$. To help quantify the reduction in this noise power by the beamformer, we assume the noise power is the same at all sensors, and equal to a scalar denoted by R_n. The noise power at the output of the beamformer is given by

$$R_{\hat{x}} = \mathbf{w}^H \mathbf{R}_n \mathbf{w} = \frac{1}{\mathbf{v}^H \mathbf{R}_n^{-1} \mathbf{v}}. \tag{2}$$

Therefore, the SNR gain due to the array, denoted as the “array gain”, is given by

$$A = R_n / R_{\hat{x}} = R_n \mathbf{v}^H \mathbf{R}_n^{-1} \mathbf{v} \equiv \mathbf{v}^H \boldsymbol{\rho}_n^{-1} \mathbf{v}. \tag{3}$$

Here, $\boldsymbol{\rho}_n$ is a normalized covariance matrix where the entries on the matrix diagonal are unity. In the case of uncorrelated noise, the array gain will be simply $A = \mathbf{v}^H \mathbf{v} = N$, where N is the number of sensors. In the case of correlated noise, the array gain could be more or less than N.

2 DIRECTIONAL SPECTRUM OF IONOSPHERIC ECHOES

To estimate $\boldsymbol{\rho}_n$ for arbitrary receive arrays, we can characterize the ionospheric echo as a spectrum of plane waves $S(\mathbf{k}, \omega)$ incident on the array, where $\mathbf{k}$ is a vector wavenumber (k_x, k_y, k_z), and ω is the wave frequency. In the case of free space, $S(\mathbf{k}, \omega)$ reduces to a two-dimensional distribution of horizontal wavenumbers $S(k_x, k_y)$ since ω is fixed at the radar carrier frequency and the vertical wavenumber k_z is equal to $k_z = \sqrt{(\omega/c)^2 - k_x^2 - k_y^2}$ by the freespace dispersion relation $\omega = c|\mathbf{k}|$. $S(k_x, k_y)$ is equivalent to the distribution of wave direction of arrival (DOA) in elevation and azimuth.

To estimate $S(k_x, k_y)$ we extend to two dimensions a method that was devised for the characterization of one-dimensional plasma turbulence using fixed probe pairs [2]. The plane wave spectrum $S(k_x, k_y)$ is related to the plane wave spatial autocorrelation function R by

$$\frac{1}{(2\pi)^2} \iint dk_x\, dk_y\, S(k_x, k_y) e^{ik_x X + ik_y Y} = R(X, Y), \tag{4}$$

where X and Y are the lags in the x and y directions, and the autocorrelation function R is defined as

$$R(X, Y) = E[\Phi^*(x, y)\Phi(x + X, y + Y)]. \tag{5}$$

Here, Φ is a wave field with random amplitude a and random phase θ:

$$\Phi(x, y) \quad = \quad a(x, y) e^{i\theta(x,y)}. \tag{6}$$

In analogy to instantaneous frequency in the time domain, we define local wavenumbers $K_x(x, y)$ and $K_y(x, y)$ in the spatial domain:

$$K_x(x, y) = \frac{\partial \theta(x, y)}{\partial x} \qquad K_y(x, y) = \frac{\partial \theta(x, y)}{\partial y}. \tag{7}$$

By the moment-generating property of the Fourier transform, we have that

$$\frac{1}{(2\pi)^2} \iint dk_x\, dk_y (ik_x)^M (ik_y)^N S(k_x, k_y) = \frac{\partial^M \partial^N}{\partial X^M \partial Y^N} R(X, Y)\Big|_{X,Y=0} \tag{8}$$

$$= E\left[a(x,y)e^{-i\theta(x,y)} \sum_{m=0}^{M} \sum_{n=0}^{N} \binom{M}{m}\binom{N}{n} \frac{\partial^m \partial^n}{\partial X^m \partial Y^n} a(x+X,y+Y) \frac{\partial^{M-m}\partial^{N-n}}{\partial X^{M-m}\partial Y^{N-n}} e^{i\theta(x+X,y+Y)} \right]_{X,Y=0} . \tag{9}$$

If the amplitude and wavenumber spatial variation is slow compared to a wavelength such that

$$\left| \frac{1}{a(x,y)} \frac{\partial^m a(x,y)}{\partial X^m} \right|, \left| \frac{1}{K_x(x,y)} \frac{\partial^m K_x(x,y)}{\partial X^m} \right| \ll |K_x^m(x,y)| \tag{10}$$

$$\left| \frac{1}{a(x,y)} \frac{\partial^n a(x,y)}{\partial Y^n} \right|, \left| \frac{1}{K_y(x,y)} \frac{\partial^n K_y(x,y)}{\partial Y^n} \right| \ll \left| K_y^n(x,y) \right|, \tag{11}$$

for all $m, n > 0$, then it can be shown that Equations (8) and (9) reduce to

$$\frac{1}{(2\pi)^2} \iint dk_x\, dk_y\, k_x^M k_y^N S(k_x,k_y) \approx E[a^2(x,y)K_x^M(x,y)K_y^N(x,y)]. \tag{12}$$

Since a, K_x, and K_y are random variables, they have a joint probability density function (PDF) $f(a^2, K_x, K_y)$:

$$\frac{1}{(2\pi)^2} \iint dk_x\, dk_y k_x^M k_y^N S(k_x,k_y) \approx \iiint da^2\, dK_x\, dK_y\, a^2(x,y)K_x^M(x,y)K_y^N(x,y) f(a^2,K_x,K_y), \tag{13}$$

and so the plane wave spectrum in the vicinity of the receive array can be written

$$S(k_x,k_y) \approx (2\pi)^2 \int da^2\, a^2(x,y) f(a^2,K_x,K_y). \tag{14}$$

Thus the wave spectrum is proportional to a power-weighted joint PDF of the local wavenumbers. This result is analogous to the time domain result that the frequency spectrum is equal to the spectrum of instantaneous frequencies when the signal modulation rate is slow compared to the signal bandwidth.

3 DIRECTIONAL SPECTRUM MEASUREMENTS

In practice, the right side of Equation (14) can be estimated by compiling a power-weighted histogram of the phase lags from antenna pairs spaced in the x and y directions. To this end, experiments were carried out on 15 Oct 2003 at an HFSWR installation at Cape Race, Newfoundland, Canada. This facility is described in general terms in [3]. For the characterization of the ionospheric signals, the facility was operated in a ship surveillance mode at 3.2 MHz, and ionospheric Spread F clutter signals were collected by monopole receive antenna pairs that were spaced 54.0 m in the x direction (approximately southeast) and 33.3 m in the y direction (approximately northeast). The data were analyzed for phase lags and power content and the results are plotted in Figure 1.

The left panel of Figure 1 shows a set of DOA measurements where zenith is the origin of the plot and the horizon is the edge of the circle. Here, K_x is the horizontal axis, and K_y is the vertical axis. This scatter plot suggests that $S(k_x,k_y)$ depends on the radial distance $\sqrt{k_x^2 + k_y^2}$ rather than k_x and k_y individually. The center and right panels are plots of the marginal distributions $S(k_x) = \int dk_y\, S(k_x,k_y)$ and $S(k_y) = \int dk_x\, S(k_x,k_y)$, which take the form of Gaussian distributions of comparable variance. These experimental results suggest that we model the power spectrum $S(k_x,k_y)$ as an uncorrelated bivariate Gaussian distribution centered at approximately zenith:

$$S(k_x,k_y) \approx \frac{2\pi}{\sigma^2} e^{-\frac{k_x^2+k_y^2}{2\sigma^2}}. \tag{15}$$

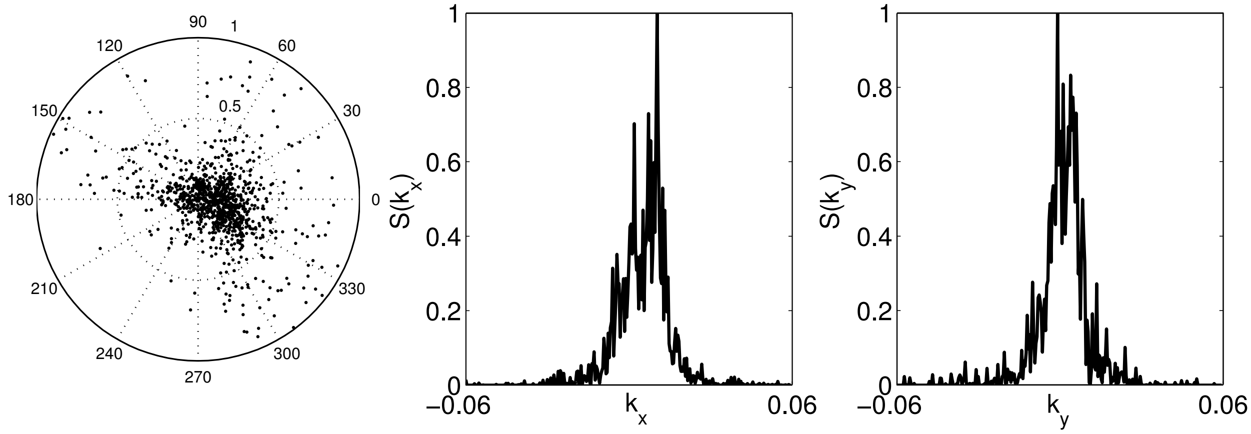

Figure 1: Left, measured DOA scatter plot. Center, measured $S(k_x)$. Right, measured $S(k_y)$.

We will retain the standard deviation σ of this distribution as a parameter in our analysis. The parameter σ is a function of the roughness of the reflecting ionospheric plasma, and describes the angular spread α of the ionospheric reflections:

$$\alpha = 2\sin^{-1}(\sigma/k). \tag{16}$$

The plots of Figure 1 suggest an angular spread α of about 16 degrees.

To compute the normalized array covariance matrix $\boldsymbol{\rho}_n$, we evaluate the spatial autocorrelation function R associated with the plane wave spectrum $S(k_x, k_y)$ at points corresponding to element locations in the receive array. The spatial autocorrelation function is given by

$$R(X,Y) = \frac{1}{(2\pi)^2}\int dk_x\, dk_y\, S(k_x,k_y)e^{ik_xX+ik_yY} = e^{-\frac{X^2+Y^2}{2/\sigma^2}}. \tag{17}$$

Thus any pair of array elements in a two-dimensional array have a correlation that decays as a Gaussian with respect to their separation in the x and y directions, where the correlation distance is $1/\sigma$. In this experiment the correlation distance is about 100 m.

4 ADAPTIVE ARRAY GAIN CALCULATIONS

We can now determine the array covariance matrix $\boldsymbol{\rho}_n$ for various array configurations by evaluating the spatial autocorrelation function R at the locations of the array elements. The adaptive array gain A for these array configurations can then be calculated according to Equation (3). This procedure has been carried out for four common array configurations, each of $N = 16$ elements and interelement spacing of $\lambda/2$. These configurations consist of (1) a 16-element linear array, (2) a cross of two 8-element linear arrays, (3) a 16-element circular array, and (4) a filled-aperture 4x4 square array. The variation in A has been calculated as a function of the azimuthal angle of the target signal with respect to array broadside, and as a function of the angular spread α of the ionosphere signals, as given by Equation (16). The results are plotted in Figure 2.

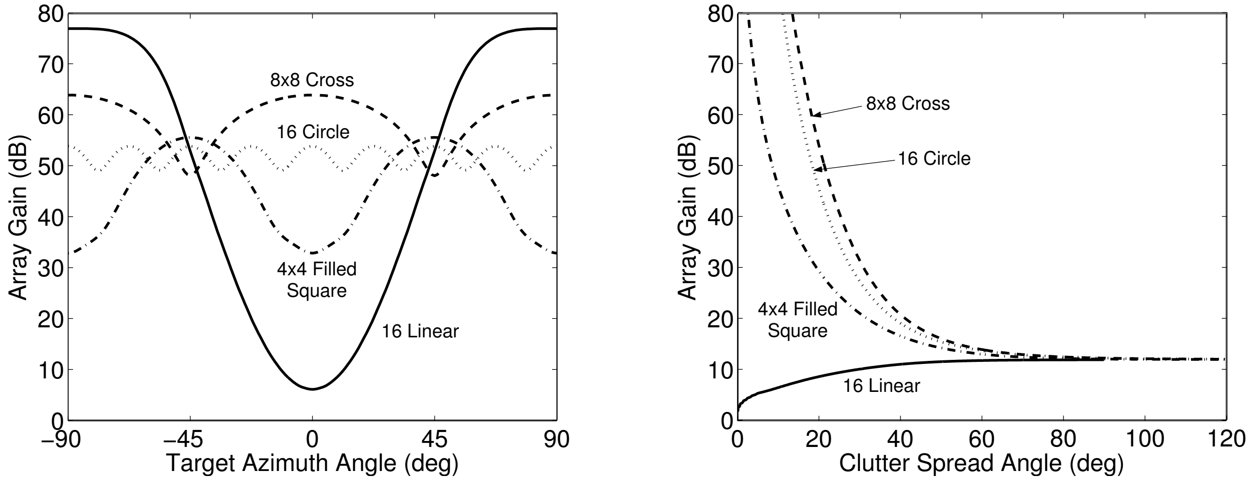

Figure 2: Left, adaptive array gain versus target azimuth angle with respect to broadside (clutter spread angle = 16 degrees). Right, adaptive array gain for broadside target versus clutter spread angle.

The results have physical interpretations as follows. The adaptive array gain for a fixed clutter spread angle (left panel of Figure 2) varies with the aperture of a projection of the array on a vertical plane oriented in the target direction. For example, when the target is broadside to the 16-element linear array, the ionospheric clutter is also broadside with respect to the array, and the array has difficulty distinguishing the low-elevation surface vessel echo from the high-elevation ionospheric echo. This effect becomes more pronounced as the clutter spread angle α goes to zero, as shown in the right panel of Figure 2. In contrast, the two-dimensional arrays have better array gains for targets near broadside due to their finite aperture projection on the vertical plane containing the target direction. The zenith array null becomes perfect when $\alpha = 0$ and the array gains for these configurations become infinite. Conversely, as α gets large, the clutter signal becomes uncorrelated noise for all array configurations, and the array gains approach the uncorrelated case of $A = N = 12$ dB.

5 REFERENCES

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